

- (1) \_\_\_\_\_ For how many positive integer values for  $k$  in the equation  $kx + 30 = 6k$  is the value of  $x$  also a positive integer?
- (2) \_\_\_\_\_ What is the sum of the greatest common divisor of 50 and 5005 and the least common multiple of 50 and 5005?
- (3) \_\_\_\_\_ How many positive factors does 30 have?
- (4) \_\_\_\_\_ What is the units digit of the product of all the odd positive integers between 10 and 110?
- (5) \_\_\_\_\_ What is the smallest prime that is the sum of four different, positive composite integers?
- (6) \_\_\_\_\_ Base prime representation of a natural number is defined using the exponents of its prime factorization as follows. Each place in a base prime represents a prime number, and it is occupied by the corresponding exponent of that prime, starting on the right side with the smallest prime number and proceeding to the left with the next largest prime number. For instance, since  $84 = 7^1 \times 5^0 \times 3^1 \times 2^2$ , then 84 would be written as 1012 in base prime. What is 225 written in base prime?
- (7) \_\_\_\_\_ What is the smallest positive integer that has exactly eight distinct positive factors?
- (8) \_\_\_\_\_ How many fractions in the form  $\frac{n}{99}$ , with  $0 < n < 99$ , are in lowest terms?
- (9) \_\_\_\_\_ The line  $5x + 6y = 27$  passes through a point whose coordinates are both positive integers. What are the coordinates of this point expressed as  $(x, y)$ ?
- (10) \_\_\_\_\_ What is the sum of the units digits of all the multiples of 3 between 0 and 50?

- (11) \_\_\_\_\_ What is the greatest common factor of  $20!$  and  $200,000$ ? (Reminder: If  $n$  is a positive integer, then  $n!$  stands for the product  $1 \cdot 2 \cdot 3 \cdot \dots \cdot (n-1) \cdot n$ .)
- (12) \_\_\_\_\_ The integer 12 has five one-digit factors, 20 has four one-digit factors, and together 12 and 20 have nine one-digit factors (some of which are repeats.) What is the total number of one-digit factors for all of the integers from 1 to 100, inclusive?
- (13) \_\_\_\_\_ Boxes are arranged in a 20-layer tower. The top layer has three boxes, and each subsequent layer has one more than twice the number of boxes in the layer above it. If  $2^n$  is the largest power of two that is a factor of the total number of boxes, what is the value of  $n$ ?
- (14) \_\_\_\_\_ How many positive three-digit integers with a 5 in the units place are divisible by 15?
- (15) \_\_\_\_\_ The sum of three consecutive prime numbers is 173. What is the largest of these numbers?
- (16) \_\_\_\_\_ The product of all the positive factors of  $6^3$  can be written in the form  $6^M$ , for some positive integer  $M$ . What is the value of  $M$ ?
- (17) \_\_\_\_\_ Three teenagers have integer ages  $x, y$ , and  $z$ , in years. If the product of their ages is 4,590 and they each have a different age, what is the sum of the three ages, in years?
- (18) \_\_\_\_\_ Mrs. Jackson baked a batch of cookies. If she makes bags of cookies with 3 cookies in each bag, 2 cookies are left over. If she makes bags with 5 cookies in each bag, no cookies are left over. If she makes bags with 8 cookies in each bag, 6 cookies are left over. What is the fewest number of cookies that Mrs. Jackson could have baked?
- (19) \_\_\_\_\_ What is the greatest common divisor of  $5!$  and  $\frac{8!}{3!}$ ? Express your answer as an integer. (Reminder: If  $n$  is a positive integer, then  $n!$  stands for the product  $1 \cdot 2 \cdot 3 \cdot \dots \cdot (n-1) \cdot n$ .)

- (20) \_\_\_\_\_ Six-digit integers will be written using each of the digits 1 through 6 exactly once per six-digit integer. How many different positive integers can be written such that all pairs of consecutive digits of each integer are relatively prime? (Note: 1 is relatively prime to all integers.)
- (21) \_\_\_\_\_ What is the maximum number of 2-inch by 2-inch by 2-inch cubes that can be placed in a box that measures 7-inches by 8-inches by 9-inches?
- (22) \_\_\_\_\_ In order, the first four terms of a sequence are 2, 6, 12 and 72, where each term, beginning with the third term, is the product of the two preceding terms. If the ninth term is  $2^a 3^b$ , what is the value of  $a + b$ ?
- (23) \_\_\_\_\_ If the pattern shown continues, what is the letter in the 2010th position?  
MATHLETEMATHLETEMATHLETE...
- (24) \_\_\_\_\_ What is the sum of the distinct prime divisors of  $11 + 11^2$ ?
- (25) \_\_\_\_\_ If  $m$  and  $n$  are positive integers and  $3m + 9m + 27n + 81n = 204$ , what is the value of the product  $mn$ ?
- (26) \_\_\_\_\_ What is the sum of the distinct prime factors of the integer 385?
- (27) \_\_\_\_\_ A jar contains over 40 marbles. Each marble is red, white or blue. If  $\frac{3}{7}$  of the marbles are red and  $\frac{2}{7}$  of the marbles are white, what is the smallest possible number of blue marbles in the jar?
- (28) \_\_\_\_\_ When all of the factors of 90 are multiplied together, the product can be expressed as  $90^k$ . What is the value of  $k$ ?
- (29) \_\_\_\_\_ Cammie has some pennies, nickels, dimes, and quarters. What is the least number of coins that she can use to make 93 cents?

- (30) \_\_\_\_\_ The integer  $A67,83B$  where  $A$  and  $B$  are the first and last digits of the integer, respectively, is divisible by 15. What is the largest possible sum of  $A$  and  $B$ ?

# Answer Sheet

Number	Answer	Problem ID
1	4 values	B2203
2	50055	20303
3	8 (factors)	5DC53
4	5	0C203
5	29	4AAD3
6	220 base prime	04CC2
7	24	AC103
8	60 fractions	5A203
9	(3,2)	CD003
10	78	3B403
11	40,000	0B103
12	281 one-digit factors	B5403
13	3	0A253
14	30 three-digit integers	11303
15	61	BC353
16	24	D1CC2
17	50 (years)	3AA53
18	110 cookies.	1A353
19	120	510D3
20	72 integers	03103
21	48 (cubes)	B1C53
22	55	C4CC2
23	A	ABBC2
24	16	CAB53
25	8	B4303
26	23	2CAD3
27	12 blue marbles	12003
28	6	3B253
29	8 coins	4A3D3
30	12	34C53

## Solutions

(1) **4 values** ID: [B2203]

We move all terms with  $k$  to one side and then factor.

$$30 = 6k - kx = k(6 - x)$$

Since  $k$  and 30 are positive,  $6 - x$  must be positive. That means  $1 \leq x \leq 5$ . There are 5 possible values for  $x$ , but not all of them may work. We have to check that 30 is divisible by  $6 - x$  so that  $k$  is an integer. We find that 30 is divisible by  $6 - x$  when  $x$  is 1, 3, 4, or 5. There is a value of  $k$  corresponding to each of the values of  $x$ , so there are 4 values for  $k$ .

(2) **50055** ID: [20303]

The prime factorization of 50 is  $2 \cdot 5^2$  and the prime factorization of 5005 is  $5 \cdot 7 \cdot 11 \cdot 13$ .

The greatest common divisor is 5 and the least common multiple is

$2 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 = 2 \cdot 5 \cdot 5005 = 50050$ . The sum of the GCD and LCM is 50055.

(3) **8 (factors)** ID: [5DC53]

A positive integer with prime factorization

$$p_1^{e_1} p_2^{e_2} p_3^{e_3} \dots$$

where the  $p_i$  are distinct prime factors and the  $e_i$  are exponents of the aforementioned prime factors has

$$(e_1 + 1)(e_2 + 1)(e_3 + 1) \dots$$

positive integer factors. Here we have  $30 = 2 \cdot 3 \cdot 5 = 2^1 \cdot 3^1 \cdot 5^1$ , so it has

$(1 + 1)(1 + 1)(1 + 1) =$ 8 $$  positive factors.

(4) **5** ID: [0C203]

Any odd multiple of 5 will end in a units digit of 5 (even multiples will end in a units digit of 0). Since all the integers we are multiplying are odd and some of them have a factor of 5, the product will be an odd multiple of 5 with a units digit of 5.

(5) **29** ID: [4AAD3]

The smallest 4 composite integers are 4, 6, 8, and 9. These sum to  $4 + 6 + 8 + 9 = 27$ .

The smallest prime greater than or equal to this number is 29. 29 can be expressed as

$4 + 6 + 9 + 10$ , so our answer is 29.

(6) **220 base prime** ID: [04CC2]

The prime factorization of 225 is  $225 = 15^2 = 3^2 \times 5^2$ . Since 2 does not divide into 225, we treat 2 as having a 0 exponent; the next two primes are 3 and 5. Thus, the answer is 220.

(7) **24** ID: [AC103]

To find the smallest positive integer with exactly four factor pairs, we want the number to be divisible by 1, 2, 3, and 4. So the number is  $1 \cdot 2 \cdot 3 \cdot 4 =$ 24.

(8) **60 fractions** ID: [5A203]

The prime factorization of 99 is  $3^2 \cdot 11$ . So for  $\frac{n}{99}$  to be in lowest terms,  $n$  cannot be divisible by 3 or 11. The possible values of  $n$  are from 1 to 98, inclusive, so there are 98 possible values for  $n$ . We can find the number of multiples of 3 and multiples of 11 and subtract from 98 to get the number of values that aren't divisible by 3 or 11. For multiples of 3, we go from 3 to 96, or  $3 \cdot 1$  to  $3 \cdot 32$ , so there are 32 multiples of 3 from 1 to 98, inclusive. For multiples of 11, we go from 11 to 88, so there are 8 multiples of 11 from 1 to 98, inclusive. We have to make sure we don't double-count the numbers that are multiples of both 3 and 11: 33 and 66. So there are  $32 + 8 - 2 = 38$  values of  $n$  that are divisible by 3 or 11. That means there are  $98 - 38 =$ 60 values of  $n$  for which  $\frac{n}{99}$  is in lowest terms.

(9) **(3,2)** ID: [CD003]

We can write  $6y = 27 - 5x$ , so since the left-hand side is divisible by 3, the right-hand side must also be. This means that  $x$  must be divisible by 3, so therefore  $x = 3$  since when  $x \geq 6$   $y$  would be negative. Therefore,  $y = 2$  and thus the point is (3, 2).

(10) **78** ID: [3B403]

We start by computing the sum of the units digits of all multiples of 3 between 0 and 30. Excluding 0, every possible digit appears exactly once as a unit digit of a multiple of 3: the set of multiples of 3 between 0 and 30 consists of the numbers 0, 3, 6, 9, 12, 15, 18, 21, 24, 27, 30. Thus, the sum of their units digits is equal to

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = \frac{9 \cdot 10}{2} = 45.$$

We must sum the units digits of the multiples of 3 between 31 and 50. The relevant multiples of 3 are 33, 36, 39, 42, 45, 48, and the sum of their units digits is  $3 + 6 + 9 + 2 + 5 + 8 = 33$ . Thus, the answer is  $45 + 33 =$ 78.

(11) 40,000 ID: [0B103]

The prime factorization of 200,000 is  $2^6 \cdot 5^5$ . Then count the number of factors of 2 and 5 in 20!. Since there are 10 even numbers, there are more than 6 factors of 2. There are 4 factors of 5. So the greatest common factor is  $2^6 \cdot 5^4 = \boxed{40,000}$ .

(12) 281 one-digit factors ID: [B5403]

Although this problem can be solved by casework, there is a more elegant method. Instead of counting how many factors each integer has, we can count how many integers have each factor, and end up with the same answer much more quickly. Every integer is divisible by 1, so there are 100 total factors that are 1. Notice that  $100 \div 1 = 100$ . The next one-digit factor is 2, and there are 50 integers between 1 and 100 that are divisible by 2, and this can be found through  $100 \div 2 = 50$ . For 3,  $100 \div 3$  is not a whole number, but since we are counting factors, simply round down to the nearest whole number, and there are 33 integers that are divisible by 3 (this can be verified by counting, or noticing that the closest multiple of 3 to 100 is 99, and  $99 \div 3 = 33$ ). Continuing in this manner, we find that there are  $100 \div 4 = 25$  numbers with 4 as a factor,  $100 \div 5 = 20$  multiples of 5, 16 multiples of 6, 14 multiples of 7, 12 multiples of 8, and 11 multiples of 9.

To finish, add all the one-digit factors together.

$$100 + 50 + 33 + 25 + 20 + 16 + 14 + 12 + 11 = \boxed{281}.$$

(13) 3 ID: [0A253]

We compute a few values and see if there exists a pattern:

- \* The first layer has 3 boxes
- \* The second layer has  $2(3) + 1 = 7$  boxes
- \* The third layer has  $2(7) + 1 = 15$  boxes
- \* The fourth layer has  $2(15) + 1 = 31$  boxes

We observe that the number of boxes in each row is one less than a power of 2, and this pattern will always hold (since  $2(2^n - 1) + 1 = 2^{n+1} - 1$ , which is one less than a power of 2). The  $n$ th layer has  $2^{n+1} - 1$  boxes. So, the total number of boxes in the 20-layer tower is

$$\begin{aligned}(2^2 - 1) + (2^3 - 1) + (2^4 - 1) + \cdots + (2^{21} - 1) &= (2^2 + 2^3 + 2^4 + \cdots + 2^{21}) - 20 \cdot 1 \\ &= 4(1 + 2 + 2^2 + \cdots + 2^{19} - 5) \\ &= 4(2 + 2^2 + \cdots + 2^{19} - 4) \\ &= 8(1 + 2 + \cdots + 2^{18} - 2)\end{aligned}$$

Notice how the value in the parentheses will always be odd (since all but the first term are even). Hence  $8 = 2^3$  is the highest power of 2 that divides the total number of boxes, and  $n = \boxed{3}$ .



(14) **30 three-digit integers** ID: [11303]

Odd multiples of 5 have a 5 in the units place, while even multiples of 5 have a 0 in the units place. Multiples of 15 are multiples of 5, so we look at how many three-digit multiples of 15 are odd to find how many have a 5 in the units place. The three-digit multiples of 15 range from 105 to 990, or  $15 \times 7$  to  $15 \times 66$ . So there are  $66 - 7 + 1 = 60$  three-digit multiples of 15. Half of them will be odd, so there are 30 odd, three-digit multiples of 15. There are 30 positive, three-digit integers with a 5 in the units place that are divisible by 15.

(15) **61** ID: [BC353]

First, we divide 173 by 3 to get  $173 \div 3 = 57.\overline{6}$ . This means that each of the three prime numbers are around  $57.\overline{6}$ . We get that 47, 53, 59, 61, and 67 are all prime numbers close to  $57.\overline{6}$ .  $53 + 59 + 61 = 173$  and the largest of these numbers is 61.

(16) **24** ID: [D1CC2]

$6^3$  is 216. Prime factorization of 216 is  $2^3 \times 3^3$ . This means each divisor of 216 will have a power of 2, and this power can be 0, 1, 2, or 3. Each divisor will also contain a power of 3, and this power can also be 0, 1, 2, or 3. Since there are 4 possible powers of 2 and 4 possible powers of 3, there are  $4 \times 4 = 16$  divisors. We can now find the positive factors of 216 like so:

$$2^0 \times 3^0 = 1$$

$$2^1 \times 3^0 = 2$$

$$2^2 \times 3^0 = 4$$

$$2^3 \times 3^0 = 8$$

$$2^0 \times 3^1 = 3$$

$$2^1 \times 3^1 = 6$$

$$2^2 \times 3^1 = 12$$

$$2^3 \times 3^1 = 24$$

$$2^0 \times 3^2 = 9$$

$$2^1 \times 3^2 = 18$$

$$2^2 \times 3^2 = 36$$

$$2^3 \times 3^2 = 72$$

$$2^0 \times 3^3 = 27$$

$$2^1 \times 3^3 = 54$$

$$2^2 \times 3^3 = 108$$

$$2^3 \times 3^3 = 216$$

The product of all of these factors is a very large and unwieldy number, and it would be wise not to work with it directly. However, we know that there are 16 factors of 216, or  $6^3$ . The factors multiply with each other to get 216, so there are 8 pairs of factors, meaning that in the product of all the factors,  $6^3$  occurs 8 times. Therefore, the product of all the factors is  $6^{3 \times 8}$ , or  $6^2 \cdot 4$ . So,  $M = 24$ . (You can verify with a calculator that  $6^2 \cdot 4$  is equal to the product of the factors).

(17) **50 (years)** ID: [3AA53]

We factor:  $4590 = 459 \cdot 10 = 51 \cdot 9 \cdot 10 = 3 \cdot 17 \cdot 3 \cdot 3 \cdot 2 \cdot 5$ .

One teen is clearly 17. The other two teens are  $3 \cdot 3 \cdot 2 = 18$  and  $3 \cdot 5 = 15$ . Thus the sum of the three ages is  $17 + 18 + 15 = \boxed{50}$  years.

(18) **110 cookies.** ID: [1A353]

Let  $n$  be the number of cookies Mrs. Jackson bakes. Looking at only the first and second statements, we see that  $n$  must be a multiple of 5, and have remainder 2 when divided by 3. The smallest number that satisfies these conditions is 5. The least common multiple of 5 and 3 is 15. Thus, 5 plus any multiple of 15 will have a remainder 0 when divided by 5 and remainder 2 when divided by 3. Thus,  $n$  is of the form  $5 + 15k$  for some non-negative integer  $k$ . The final statement says that when  $n$  is divided by 8, there is a remainder of 6. Listing out the first several possible numbers of the form  $5 + 15k$ , we get

$$5, 20, 35, 50, 65, 80, 95, 110, \dots$$

Testing each of these numbers to find their remainders when divided by 8, we see that 110 is the lowest number with remainder 6. Thus, the lowest possible number of cookies that Mrs. Jackson could have baked is 110 cookies.

(19) **120** ID: [510D3]

First, we factor  $5!$  into primes:

$$\begin{aligned} 5! &= 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \\ &= 2^3 \cdot 3 \cdot 5. \end{aligned}$$

Then, we factor  $\frac{8!}{3!}$ .

$$\begin{aligned} \frac{8!}{3!} &= 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \\ &= 2^6 \cdot 3 \cdot 5 \cdot 7. \end{aligned}$$

We can find the greatest common factor by taking the lowest exponent of each common prime factor. We get  $2^3 \cdot 3 \cdot 5 =$ 120.

(20) **72 integers** ID: [03103]

We write down all pairs of relatively prime integers: (1,2), (1,3), (1,4), (1,5), (1,6), (2,3), (2,5), (3,4), (3,5), (4,5), (5,6). However, since we want all of the even integers separated because no even numbers are relatively prime, we want the six digit number to be three sets of pairs that have one odd and one even number. This leaves the odd-even pairs of (1,2), (1,4),(1,6),(2,3), (2,5), (3,4),(4,5), (5,6).

We want to find how many ways we can have 3 sets such that every number only appears once. We see that 3 only appears in 2 pairs. For the first pair (2,3), we can't have 2 or 3 in any of the other two pairs. This leaves (1,4), (1,6), (4,5), (5,6). From this we get two ways: (1,4),(5,6) and (1,6),(4,5).

Next, for the pair (3,4), we can't have 3 or 4 in either of the other two pairs, which leaves (1,2), (1,6), (2,5), (5,6). We get two ways: (1,2),(5,6) and (1,6),(2,5).

Therefore, we get that there are  $2 + 2 = 4$  ways to get three pairs. First, we have the number to alternate odd, even with odd going first. We get that for each of the four ways, there are  $3! = 6$  ways to permute the three pairs. Since there are four ways, we get  $4 \cdot 6 = 24$ . We multiply that by 2 for the case where we have the number to alternate even, odd, with even going first. This gives us  $24 \cdot 2 = 48$ .

Next we get that we could have *even, odd, odd, even* given that the two odds are also relatively prime. The odd numbers that are relatively prime are (1,3), (1,5), (3,5). Since we could flip these numbers, we get 6 ways to have those two odd numbers in the middle. For each of these 6 ways, we get that there are 2 different even number pairs that go with the odd number, so we have  $6 \cdot 2 = 12$ . Since there are three pairs, there are also 2 ways to add the other pair (we can have (even odd) even odd odd even or even odd odd even (odd even)), where the third pair was in the parenthesis. This gives us  $12 \cdot 2 = 24$  ways.

Adding all of these cases together, we get that there are  $48 + 24 = \boxed{72 \text{ numbers}}$ .

(21) **48 (cubes)** ID: [B1C53]

7-inches contains 3 2-inch pieces; 8-inches contains 4 2-inch pieces, and 9-inches contains 4 2-inch pieces, so extra space aside, the box contains  $3 \cdot 4 \cdot 4 = \boxed{48}$  cubes.

(22) **55** ID: [C4CC2]

Let us write out the prime factorizations of each term of the sequence. We have:

$$2 = 2^1 \cdot 3^0$$

$$6 = 2^1 \cdot 3^1$$

$$12 = 2^2 \cdot 3^1$$

$$72 = 2^3 \cdot 3^2.$$

Notice that the exponent of the power of 2 on each term is equal to the sum of the exponents of the power of 2 on the previous two terms, and similarly for 3. Indeed, if two consecutive terms of the sequence are given by  $2^{e_2} \cdot 3^{e_3}$  and  $2^{f_2} \cdot 3^{f_3}$ , then their product is  $2^{e_2+f_2} \cdot 3^{e_3+f_3}$ . It follows that the exponents of the 2 and 3 satisfy the Fibonacci sequence. After inspection, the ninth term is equal to  $2^{F_9} \cdot 3^{F_{10}}$ , where we calculate  $F_9 = 34$ ,  $F_8 = 21$ . Thus, the answer is  $34 + 21 = \boxed{55}$ .

(23) **A** ID: [ABBC2]

The pattern repeats with every 8 letters, so we can determine the letter in the  $n$ th position by finding the remainder when  $n$  is divided by 8. Since 2008 is divisible by 2 and 4, we know it is a multiple of 8 (so it has a remainder of 0 when 2008 is divided by 8). That means 2010 has a remainder of 2 when divided by 8. The letter in the 2010th position will be the same as the letter in the second position of the pattern, which is  $\boxed{A}$ .

(24) **16** ID: [CAB53]

We have  $11 + 11^2 = 11(1 + 11) = 11(12) = 11 \cdot 2^2 \cdot 3$ , so the sum of the distinct prime divisors is  $11 + 2 + 3 = \boxed{16}$ .

(25) **8** ID: [B4303]

We can first simplify the equation by dividing all terms by 3.

$$3m + 9m + 27n + 81n = 204 \Rightarrow$$

$$m + 3m + 9n + 27n = 68 \Rightarrow$$

$$4m + 36n = 68 \Rightarrow$$

$$m + 9n = 17$$

Since  $n$  is a positive integer and  $9(2) > 17$ , the only possible value for  $n$  is 1. That means  $m = 17 - 9 = 8$ . The product  $mn = \boxed{8}$ .

(26) **23 ID: [2CAD3]**

Factoring, we get

$$385 = 5 \cdot 77 = 5 \cdot 7 \cdot 11$$

Thus, the sum of the distinct prime factors is  $5 + 7 + 11 = 12 + 11 = \boxed{23}$ .

(27) **12 blue marbles ID: [12003]**

If  $n$  is the number of marbles, then  $\frac{2n}{7}$  must be an integer, since it is the number of white marbles, so  $n$  is divisible by 7. Thus  $n$  is at least 42 (and in fact, this number works), so the number of blue marbles is  $(1 - \frac{3}{7} - \frac{2}{7})n = (\frac{2}{7})42 = (2)(6) = \boxed{12}$ .

(28) **6 ID: [3B253]**

$90 = 2 \cdot 3^2 \cdot 5$ , so has  $(1 + 1)(2 + 1)(1 + 1) = 12$  factors. We can group these factors into 6 pairs such that each pair multiplies to 90. Hence, all the factors of 90 multiply to  $90^6$  and we have  $k = \boxed{6}$ .

(29) **8 coins ID: [4A3D3]**

If we want to use as few coins as possible, we will want to use as many large coins as possible. The largest coins is a quarter, worth 25 cents. The most quarters Cammie can have is 3 because  $93 \div 25 = 3\text{R}18$ . Thus, after 3 quarters, we still have 18 cents left. The next largest coin is a dime, worth 10 cents. Because  $18 \div 10 = 1\text{R}8$ , Cammie can have 1 dime. Thus, we have 8 cents left. Nickels are next, worth 5 cents each. Because  $8 \div 5 = 1\text{R}3$ , Cammie can have 1 nickel. Finally, we have 3 cents remaining. The last coin is a penny, worth 1 cent each. Because  $3 \div 1 = 3\text{R}0$ , Cammie has 3 pennies, and all of the cents are accounted for. Thus, the minimal combination of coins is 3 quarters, 1 dime, 1 nickel, and 3 pennies, for a total of  $\boxed{8}$  coins.

(30) **12 ID: [34C53]**

For a number to be divisible by 15, it must be divisible by both 3 and 5.

For the number to be divisible by 3, the sum of its digits must be a multiple of 3. Thus  $A + 6 + 7 + 8 + 3 + B = 24 + A + B$  must be a multiple of 3. Since 24 is already a multiple of 3,  $A + B$  must be a multiple of 3.

For the number to be divisible by 5, its last digit must be a multiple of 5. Thus we have  $B = 0$  or  $B = 5$ . If  $B = 0$ , then  $A$  is a multiple of 3, the largest single-digit value of which is 9, so  $A + B = 9 + 0 = 9$  in this case. If  $B = 5$ , then  $A$  must be one more than a multiple of 3, the largest single-digit value of which is 7, so  $A + B = 5 + 7 = 12$  in this case. Thus the largest possible sum of  $A$  and  $B$  is  $\boxed{12}$ .